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SOLVING THE DILEMMA OF CONTRADICTORY GOALS

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Abstract. Coupled problems in engineering inevitably lead to contradictory goals for single quality criteria. Applying numerical optimization to find the best solution requires the definition of an objective function based upon these criteria as a measure of merit for the whole design. It will be shown that the usual approach of a weighted sum is improper and unreliable. Also the consideration of the complete Pareto front is not only no solution to this problem, it usually requires a not maintainable amount of calculations. Therefore, a new approach for the definition of objective functions is proposed, which solves the dilemma of contradictory goals by finding an appropriate compromise based upon the intuition.

1 INTRODUCTION

Coupled problems, and multidisciplinary optimization is one of them, often lead to contradictions with respect to the goals. These contradictions do not only occur in multidisciplinary cases, e.g. low aerodynamic drag and a wing as thick as possible for an aircraft, but also in simpler cases like multipoint designs. This results in the problem to obtain an overall appropriate measure of merit for the design, instantiated by a so called objective function. Usually, this function is set up as a weighted sum of all conceived criteria. While this approach seems to be simple and obvious, it comprises a severe amount of arbitrariness concerning the choice of usually applied weighting factors. Furthermore, if constraints for some criteria have to be considered, assuming the application of optimization algorithms which are not capable to handle them directly, this is only possible by the introduction of so called penalty functions in order to worsen the objective function value artificially in case of the violation of constraints. These penalty functions are usually quadratic, i.e. nonlinear, with the effect of an unforeseeable distortion of the solution space topography. After all, it can additionally be shown that not all possible combination of optimal criteria is possible with this approach.

An alternative approach is the consideration of the so called Pareto front. This constitutes the non-dominating solutions, meaning that the improvement of one criterion leads to a degradation of the other. Historically, this is the usual way considering criteria when evolutionary or genetic algorithms are applied, because the huge amount of objective function evaluations simply provides this front. On the other hand, there exists no distinct solution.

Users need to choose one solution out of all Pareto optimal ones, leading again to some kind of arbitrariness. Furthermore, if this may be possible in the case of only two criteria, it will become impossible for more than three, because the Pareto front becomes a hyper-dimensional surface, which cannot be imagined by humans. To handle this problem, so called decision making tools respectively algorithms have been developed, but there exists no prove for their correctness or applicability. Therefore, a new approach for an objective function, based upon the principles of Fuzzy Logic [4], is proposed, which simplifies the decision finding process significantly and leading to a distinct measure of merit for each solution. In contrast to the mentioned decision finding tools, this constitutes a real objective function.

In the following, starting with a simple example to demonstrate the difficulty in setting up an appropriate objective function based upon weighted sums, and to understand the meaning of the solution space topography, the proposed new approach will be discussed in relation to the above mentioned current procedures. Finally, a practical application to an eight criteria optimization problem will show the simplicity and advantages compared to using weighted sums.

2 SOLUTION SPACE TOPOGRAPHIES

Numerical Optimization algorithms usually need some kind of exact measure for each single solution in order to vary the design variables for an improvement. This is accomplished by setting up an objective function. However, a function means defining a surface in hyper-dimensional space. Having two criteria leads to a surface in three dimensions like a landscape. Within this landscape one can find the highest and lowest points, which are the optima. This means, by defining an objective function, the location of the optimum is also predefined, although unknown a priory. The following simple example shows this relation for only two criteria, making it possible to visualize the landscape. Having more than two criteria worsens the problem and leads to unsolvable arbitrariness.

The basis of this optimization problem is a beam with length L shown in Fig. 1, loaded by a force P and having the design parameters width w and height h. The goal is to maximize the benefit in terms of the usable length L while minimizing the costs represented by the volume V, calculated from equation (1). Assuming some kind of allowable stress $\sigma_{allowed}$ of the material, which should not be exceeded by the maximum stress occurring at the left mounting location, the resulting maximal length can be calculated based upon the geometric moment of inertia from equation (2).

$$V = L\, w\, h \tag{1}$$

$$L = \sigma_{allowed} \cdot w \cdot h^2 / (6 \cdot P) \tag{2}$$

$$\min f = w_1 V - w_2 L \tag{3}$$

The corresponding objective function, built as a weighted sum with the weighting factors w_1 and w_2, is shown in equation (3). However, it is unknown which value these factors should have. In order to demonstrate the influence of this choice, three slightly different combinations, shown in Tab. 1, were used to calculate the solution space from a full factorial search. The corresponding topographies are depicted in Fig. 2.

Table 1: Weighting Combinations for the Beam Objective Function

	Case 1	Case 2	Case 3
V	**50%**	**60%**	**40%**
L	**50%**	**40%**	**60%**

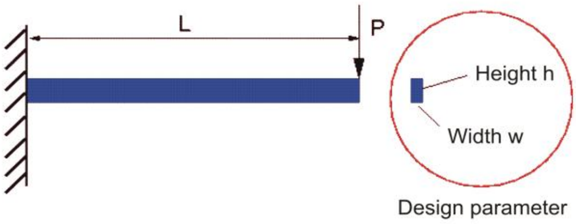


Figure 1: Beam optimization example

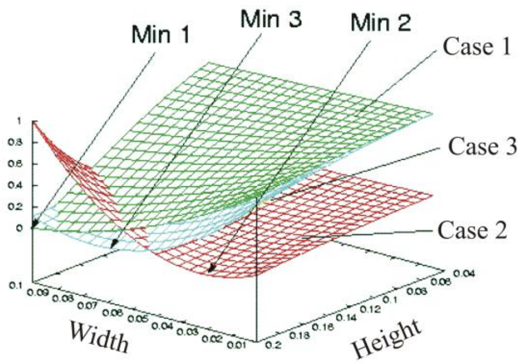


Figure 2: Beam solution spaces for different weighting factors

Different objective functions with varying weighting factors, more or less arbitrarily chosen, obviously lead to different results. Changing the weighting factors respectively the objective function means, changing the shape of the solution space, predetermining the possible optima with the result of different solutions. While this example is very simple, in practice the topographies look much more complex and non-smooth like in Fig. 3. Even in this two-dimensional case for the aerodynamic drag of an airfoil, calculated by an iterative code based upon the full potential theory, for the design parameters camber and thickness, it is obvious, that finding an optimum is not trivial within this landscape. The plateau and the peaks are due to not converging solutions, something that is very likely to occur during an optimization.

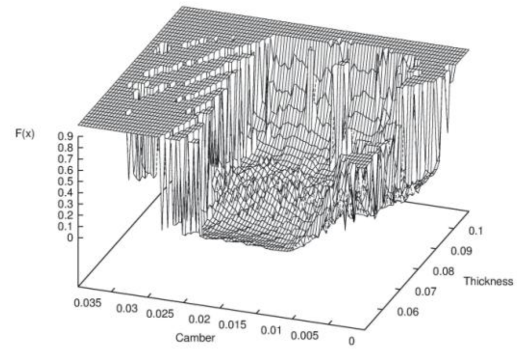


Figure 3: Solution space topography for the aerodynamic drag of an airfoil, depending on camber and thickness, calculated by a code based upon the full potential theory

3 PARETO FRONTS AND WEIGHTED SUM OBJECTIVE FUNCTIONS

In the case of multi-objective optimization no single objective can be maximized or minimized without the compromise of deteriorating another. This is known as the so called Pareto front, which includes all these solutions. To get a hands-on example, the following two objectives may serve. For this simple example we suppose, that one objective will be minimized with a decreasing design parameter, while the other one improves with an increasing one. A possibility for such a scenario are the functions from equation (4) and (5) within the interval of [0:1], Fig. 4.

$$f_1 = 1 - 2x + x^2 \tag{4}$$

$$f_2 = \sqrt{x} \tag{5}$$

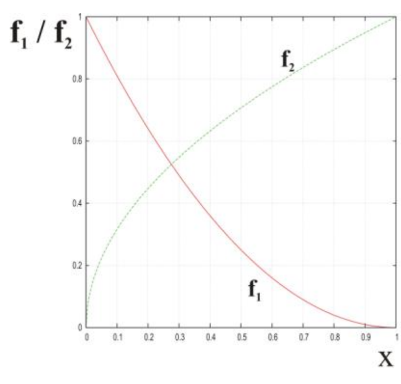


Figure 4: Competing objectives from equation (4) and (5) within the interval [0:1]

Plotting function f_2 depending on f_1 yields the Pareto front, Fig. 5, i.e. all possible compromise solutions, which in this case exist of a convex and a concave area. Now we assume to find one of these compromises by the application of a weighted sum, equation (6). Trying to incorporate this into Fig. 5 requires transposing it to equation (7). This is a linear equation with a negative gradient of w_1/w_2. Fig. 6 shows several lines with different slopes. A minimization of f means a parallel translation to the left. An equal weighting leads to the bottom right point of the front, while choosing $w_1 > w_2$ some points within the convex region can be reached. The boundary in the right picture of Fig. 6 is found for $f_1 = 1/9 = 0.111$, while the convex part ends at $f_1 = 0.444$ where the curvature changes. This means, no point on the concave part of this Pareto front and also many points on the convex part, overall about *75%*, are reachable by this kind of objective function.

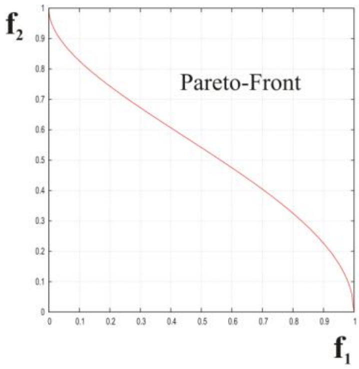


Figure 5: Pareto front for the objectives from equation (4) and (5) within the interval [0:1]

$$\min f = w_1 f_1 + w_2 f_2 \tag{6}$$

$$f_2 = f / w_2 - f_1 w_1/w_2 \tag{7}$$

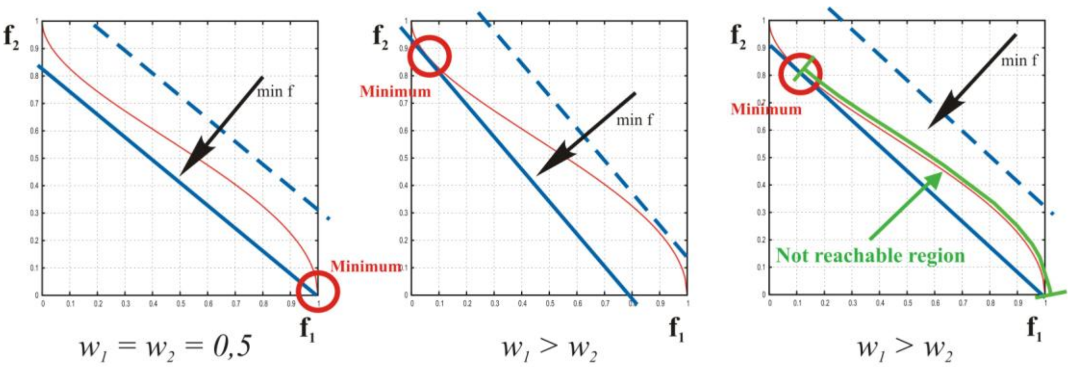


Figure 6: Pareto front compared to weighted sum objective functions

Therefore, weighted sums cannot be an appropriate approach for the description of optimization goals. Nobody ever can tell, whether the hyper-dimensional Pareto front consists of concave parts, which may be missed. Thus, another way of defining optimization goals is necessary.

4 TRANSFORMING APPRAILSALS INTO DISTINCT MEASURES

The most difficult part when applying Numerical Optimization is to find an appropriate measure of merit for each single design, i.e. to constitute the objective function. This is particularly true for cases with multiple and in general competing criteria. It seems to be self-evident to decide whether something is good or bad, but unfortunately things are never only good or bad in an absolute sense. They are more or less good or bad. What looks like a problem, adheres the solution in itself. In workaday life people need to find answers to questions like *“Does a 27 year old person belong to the group of about 30 year old people?”*. The astonishingly simple answer is “*To some extent”*, and the only question left is how to quantify this answer. Here the theory of Fuzzy Logic from Zadeh [4] comes into play. Assuming that one knows the theoretical best solution and is able to quantify an actual solution compared to that, it would be possible to specify its quality. This is the basis of Fuzzy Logic objective functions (FLOFs).

Applying Fuzzy Logic to objective functions consists of three parts:

1. Based upon the assumption that desirable, tolerable and unacceptable solutions exist, the determination of a relative quantity concerning the membership to each of these classes
2. Setting up logical rules describing the interdependencies of criteria qualities, e.g. if *criterion 1* is desirable and *criterion 2* is unacceptable, then the overall solution is unacceptable
3. Based upon the aforementioned logical rules finding some kind of compromise

The first part consists, in the terminology of Fuzzy Logic, of the constitution of so called membership functions, describing the degree of membership. In this case, a linear dependency is sufficient and yields, for the length and volume of the above example of the beam defined within the Software *CAOne*® [1], classes depicted in Fig.7. The only necessary information is an actual respectively current or starting value of the criterion and the direction for improvement.

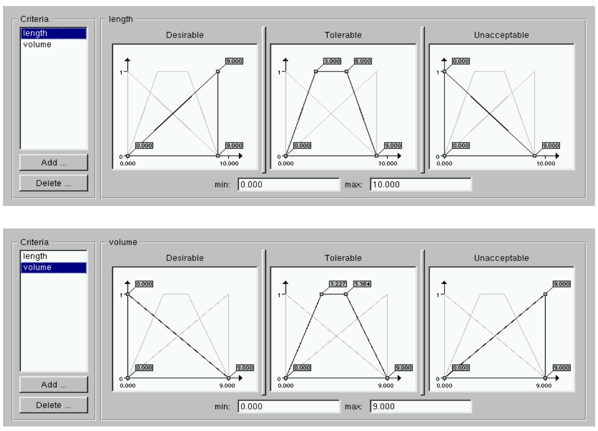
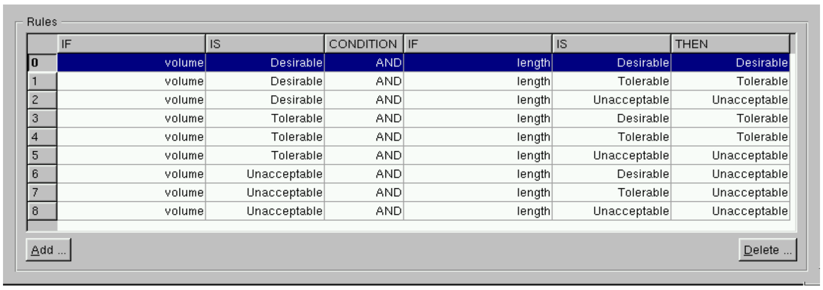


Figure 7: Classes for desirable, tolerable and unacceptable solutions for the length and volume of the beam

The second part, setting up logical rules for calculating the consequences, may look like in Fig. 8. The reader may note that it is not necessary to declare all possible rules. From experience, only the ones that are of interest should be defined. Also, the obvious rules like “*If* *the volume is desirable than the solution is desirable*" are already available within the aforementioned software. The result of each of these rules in case of an AND-condition is, comparable to binary logic, the intersection of the considered classes, meaning the minimum of the two grades of membership.


Rules

	IF	IS	CONDITION	IF	IS	THEN
0	volume	Desirable	AND	length	Desirable	Desirable
1	volume	Desirable	AND	length	Tolerable	Tolerable
2	volume	Desirable	AND	length	Unacceptable	Unacceptable
3	volume	Tolerable	AND	length	Desirable	Tolerable
4	volume	Tolerable	AND	length	Tolerable	Tolerable
5	volume	Tolerable	AND	length	Unacceptable	Unacceptable
6	volume	Unacceptable	AND	length	Desirable	Unacceptable
7	volume	Unacceptable	AND	length	Tolerable	Unacceptable
8	volume	Unacceptable	AND	length	Unacceptable	Unacceptable

Add ... Delete ...

Figure 8: Logical rules for the length and volume of the beam

The last step consists of the so called defuzzification, which means calculating an overall measure of merit for the results of the consequences. This is accomplished by calculating a compromise, again using membership functions for desirable, tolerable and unacceptable results. Fig. 9 shows the applied approach. Mathematically, it is again a weighted sum for all consequences, with the weight being the center of gravity (also called Center of Moment CoM method) of the classes respectively triangles shown in equation (8), which is the one that any optimization strategy will minimize. Here, the memberships of the consequences are denoted with s_k while x_{ck} means the location of the CoM of the respective solution class, i.e. *1/6* for the desirable, *1/2* for the tolerable and *5/6* for the unacceptable class.

$$f = \frac{\sum_k x_{s_k} s_k}{\sum_k s_k} \tag{8}$$

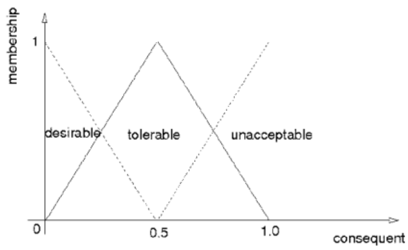


Figure 9: Classes for the desirable, tolerable and unacceptable solutions

Applying this approach to the beam example yields the solution space shown in Fig. 10. In this case it becomes obvious that the goal was to improve both criteria to the same extend, which was not clear before by applying just a weighted sum with unknown weights.

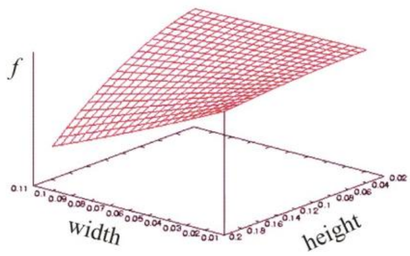


Figure 10: Solution space topography for the beam example applying a fuzzy logic objective function

5 UNDERSTANDING THE FUZZY LOGIC APPROACH

The first step in using FLOFs is the constitution of membership functions. In the case of linear dependencies this becomes a linear equation, e.g. $s_{Vd} = 1 - V/9$ and $s_{Vu} = V/9$ for the desirable or unacceptable classes of the volume in Fig. 7. Concentrating only on the desirable solutions would transform a concave Pareto front into a convex one, but also vice versa. Considering just the mere desirable solutions, without the tolerable and unacceptable ones and without any interdependency between the criteria, would yield a simple weighted sum following equation (8), which is not comparable to equation (6). Thus, it is obvious that the additional classes and the logic rules are the key ingredient for this approach. As an example, the introduction of the rule "*If Volume is desirable and Length is desirable than the solution is desirable*" would lead to equation (9), where s_V and s_L constitute the grades of membership for the desirable, tolerable and unacceptable classes respectively, while x_d, x_t and x_u are the above mentioned values for the CoM-method.

$$f = \frac{x_d \left(s_{V_d} + s_{L_d} + min\left(s_{V_d}, s_{L_d}\right)\right) + x_t \left(s_{V_t} + s_{L_t}\right) + x_u \left(s_{V_u} + s_{L_u}\right)}{s_{V_d} + s_{L_d} + s_{V_t} + s_{L_t} + s_{V_u} + s_{L_u}} \tag{9}$$

Keeping in mind that for s_V and s_L the functions for the memberships have to be introduced, and here we are only considering one single rule leading to one *min*-term (AND-condition), it becomes clear that these functions do not represent a linear line like in Fig. 6. Of course, this function consists of linear parts, but at least the division by the membership functions leads to a nonlinear curve. This makes it possible to reach even the concave parts of Pareto fronts.

In order to demonstrate this capability we will get back to the example of section 3. Let the functions f_1 and f_2 from equations (4) and (5) represent two qualities or criteria for an optimization problem, which tries to minimize both at the same time. Furthermore, it is supposed that initial solutions exist, e.g. $f_1 = 0.9$ and $f_2 = 0.3$, and any improvement is welcome. This would yield the membership functions in Fig. 11. Applying absolutely no rule, which is similar to define the obvious one like "*If f_1 is desirable and f_2 is desirable than the solution is desirable*", yields a result within the convex region, Fig. 12, but already beyond the boundary of the weighted sum. On the other hand, just introducing the rule "*If f_1 is desirable and f_2 is unacceptable than the solution is unacceptable*" finds the result on the concave part of the Pareto front, Fig. 13. This solution is impossible to find using a standard weighted sum, although it is a valid one. Fig. 14 shows the development of weighted sum based objective functions for different weighting factors in comparison to the Fuzzy Logic approach. The unreachable region is tremendously large.

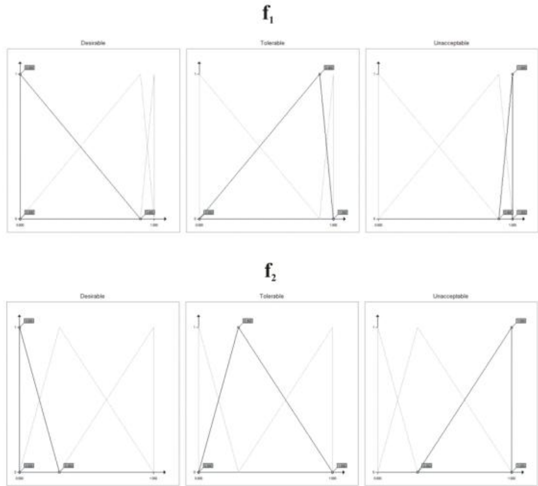


Figure 11: Membership functions

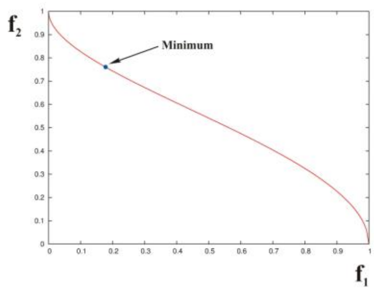


Figure 12: Minimum without any rules

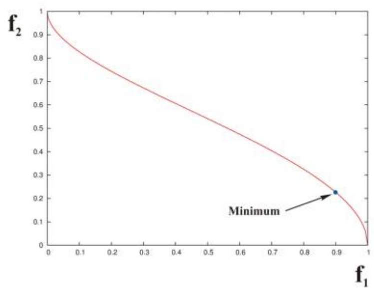


Figure 13: Minimum with one simple rule

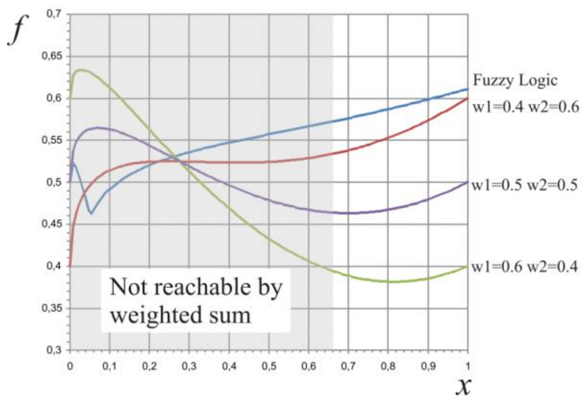


Figure 14: Comparison of different objective functions

Another effect of the application of this approach is the smoothing of the solution space topography. Fig. 15 shows on the left side a solution space which is characterized by peaks, plateaus and shallow gradients. Calculating the solution space based upon the proposed Fuzzy Logic approach yields the right picture, which shows an extremely smoothed topography. From the point of view of an optimization algorithm it is much simpler to find the optimum within this topography, particularly because the minimum has been intensified. Details concerning this can be found in [3]. This leads to a fewer number of necessary iterations and therefore additionally saves computing time.

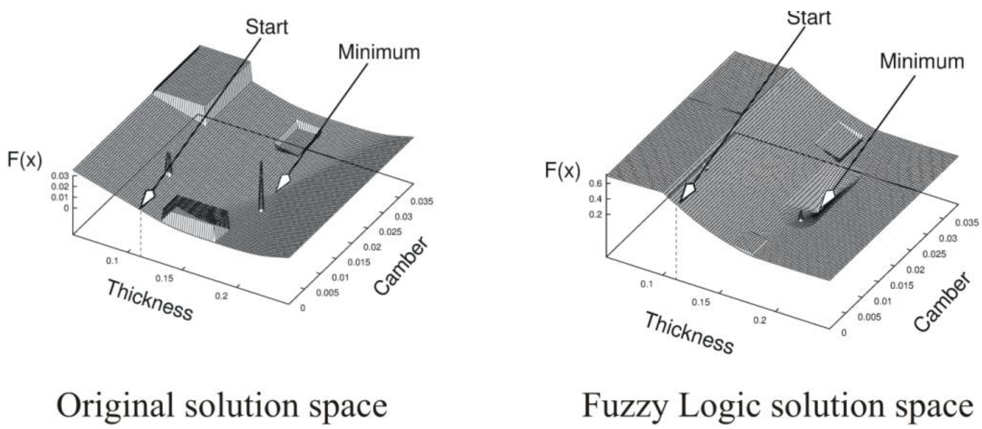


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Figure 15: Smoothing of solution space topographies

6 MULTI-CRITERIA EXAMPLE

In order to show the advantages of the proposed approach in practice, a multi-criteria example was chosen. It consists of the design of an aircraft wing for a transonic transport aircraft. This wing is set up from three lofted airfoils, each described by five design parameters, Fig. 16. The design criteria are as follows:

1. Minimum aerodynamic drag in three design points (Mach numbers / lift coefficients)
2. Minimum structural weight (depending on airfoil / wing thickness, contradictory to point 1)
3. Sufficient fuel volume for a flight mission (wing volume, contradiction to point 1)
4. Best possible low-speed properties (airfoil properties contradictory to point 1)

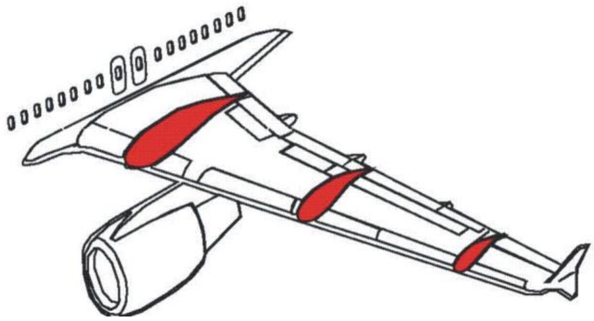

Figure 16: Aircraft wing and airfoils

These criteria are formulated in the usual linguistic manner in order to demonstrate the advantage of the FLOF approach. The modeling of all the qualities is not part of this work and can be found along with every other detail and result in [2]. All together, the designer is faced with eight criteria to be assed (three airfoil drags, the wing weight, the fuel volume and three airfoil lifts). Now it is up to the reader to give eight weighting factors for an objective function based upon a weighted sum. Obviously, this is not possible and results in a trial and error process. For this example, three different weighting combinations were chosen from experience. On the other hand, if we have an initial solution and values for each criterion, than we easily can describe the direction of improvement and quality interdependencies. The result of an optimization based upon the three weighting combinations and a FLOF is depicted in Fig. 17. The objective function values are normalized in order to make them comparable. Clearly, the Fuzzy Logic approach reaches the minimum in less iterations and the solution is better (lower values), while as expected the weighted sums lead to different results depending on the weighting values due to the deformation of the solution space topography. The FLOF leads to a decrease of the wing weight from *91t* to *60t* compared to about *83t* for the weighted sums. The same superior improvement can be found for all other criteria [2]. All together, applying the Fuzzy Logic for objective functions leads to an acceleration in convergence and better results compared to the usual approach. Above all, it is much simpler to set up and much more reliable in the sense of transforming appraisals into an exact mathematical measure of merit.

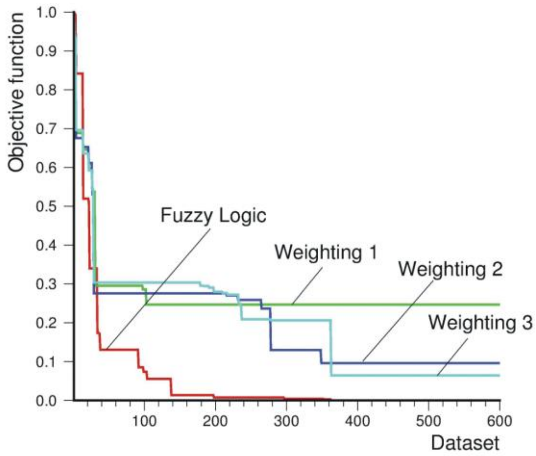

Figure 17: Convergence history for different objective functions

7 SUMMARY

Coupled problems in engineering naturally lead to contradictory criteria for each design quality and need to be assessed in relation to each other for a single goal. This is accomplished by setting up an objective function, which spans a hyper-dimensional surface in n-dimensional space. Usual approaches using weighted sums are improper, because they suffer from arbitrary choices for the weighting factors with the corresponding change in the solution space topography and, therefore, the definition of the possible optimum. Furthermore, it is impossible to describe any conceivable combination of criteria with this approach. The alternative of a so called Pareto-optimization leads to an assessment of all non dominating solutions, which is extremely difficult in the case of more than three criteria and, furthermore, shifts the procedure of the formation of opinion downstream. A new approach based upon Fuzzy Logic has been proposed, which solves all the mentioned problems and additionally yields a simple and reliable definition of design goals, even in very complicated cases. As a side effect the solution space topography is smoothed and leads to a fewer number of iterations, thus saving time and money.

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